

Studies on the Morphological Associative Memory

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1. Introduction

In 1998, a novel class of artificial neural networks, called morphological neural networks, has been introduced. In this new neural network, the operations of multiplication and addition are replaced by addition and maximum (or minimum), respectively. By taking the maximum (or minimum) of sums instead of the sum of products, morphological network computation is nonlinear before possible application of a nonlinear activation function. Now the main emphasis of the research of morphological neural network is on morphological associative memories, because we can examine the computing and storage capabilities of morphological associative memories and discuss differences between morphological models and traditional semi-linear models by this application of morphological associative networks.

In contrast to traditional associative memories, morphological associative memories converge in one step. Thus, convergence problems do not exist. Morphological analogues to the Hop-field net not only proved to be far more robust in the presence of noise but have also strong storage capacity for perfect inputs. Although the morphological associative memory was introduced, we still don't know the properties of it very clearly. In our study, we found out the condition and the formula of the perfect recall of this memory and discuss the properties of morphological associative memory deeply and clearly in order to open a new field of the application of morphological neural network.

2. Results and Discussions

The condition of perfect recall for MAN:

1. The kernel of the pattern, which is recalled, should not be damaged by the random noise
2. The input pattern with noise should not include more than one kernel of stored patterns

The formula of the perfect recalling rate of MAM

Suppose that n patterns (P_1, P_2, \dots, P_n) are stored in the morphological neural network and the fire bits of their kernels are (K_1, K_2, \dots, K_n), respectively. For pattern P_γ , the fire bits of KOP and KIP of it are KIP_γ^ξ and KOP_γ^ξ , respectively. $\gamma \in \{1, 2, \dots, n\}$ and $\xi \in \{1, 2, \dots, n\}$, $\gamma \neq \xi$. So R_γ , the Recalling Rate of Pattern P_γ , should be:

$$R_\gamma = \sum_{i_\gamma^1=0}^{KOP_\gamma^1} \sum_{j_\gamma^1=0}^{KIP_\gamma^1} \sum_{i_\gamma^2=0}^{KOP_\gamma^2} \sum_{j_\gamma^2=0}^{KIP_\gamma^2} \cdots \sum_{i_\gamma^\xi=0}^{KOP_\gamma^\xi} \sum_{j_\gamma^\xi=0}^{KIP_\gamma^\xi} \cdots \sum_{i_\gamma^n=0}^{KOP_\gamma^n} \sum_{j_\gamma^n=0}^{KIP_\gamma^n} (T)/C_p^N$$
$$T = \begin{cases} C_{KOP_\gamma^1}^{i_\gamma^1} C_{KIP_\gamma^1}^{j_\gamma^1} \cdots C_{KOP_\gamma^\xi}^{i_\gamma^\xi} C_{KIP_\gamma^\xi}^{j_\gamma^\xi} \cdots C_{KOP_\gamma^n}^{i_\gamma^n} C_{KIP_\gamma^n}^{j_\gamma^n} C_{p-K_\gamma-\sum_\xi K_\xi}^{\sum_\xi (i_\gamma^\xi + j_\gamma^\xi)} & \text{if condition} \\ 0 & \text{otherwise} \end{cases}$$

Condition :

$((i_\gamma^1 = KOP_\gamma^1) \text{and} (j_\gamma^1 = 0)) \text{or...or} ((i_\gamma^\xi = KOP_\gamma^\xi) \text{and} (j_\gamma^\xi = 0)) \text{or...or} ((i_\gamma^n = KOP_\gamma^n) \text{and} (j_\gamma^n = 0)) = F$.

R_γ : perfect recalling rate of pattern P_γ ; N : the distorted bits of pattern P_γ ; p : the size of the key pattern P_γ .

3. Conclusions

After we conclude the formula of the perfect recalling rate, we can calculate the perfect recalling rate of the any kernel of the stored patterns easily. However, we must point out that the result that we present at here just is based on the morphological associative memories for Binary pattern. The best method for gray-scale pattern hasn't been found yet. There are a lot of works about morphological associative memories that need us to do in the next study.