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Numerical Solution of Wave Equation with Non-Local Conditions

Chohan Muhammad IKHLAQ

1. Introduction

This thesis mainly comprises the finite difference methods for parabolic and hyperbolic differential equations for one dimensional space with non-local condition and analysis of numerical solution of partial differential equations by different finite difference methods.

2. Results and Discussion

We give firstly the introduction of partial differential equations and classification of linear equations in two independent variables. It also describes that initial and boundary conditions for parabolic, hyperbolic and elliptic differential equations, so the combination of partial differential equations and initial and boundary conditions lead to a well-posed problem. Some descriptions about local and non-local boundary conditions and Taylor series expansion which is very important tool to drive the finite difference approximations are present in this chapter. It also contains the solution of one dimensional heat equation and wave equation by separation of variables

Next, we discuss finite difference approximations to the derivatives, explicit and implicit methods. It provides a practical overview of numerical solution to the heat equation using the finite difference method. The forward time centered space, the backward time centered space and Crank-Nicolson schemes are developed and applied to a simple problem involving the one-dimensional heat equation. It also introduces the heat equation which is the example used to illustrate most of the various finite difference techniques. For some initial and boundary conditions, analytical solution can be found for this particular equation and its solution is useful for comparison, in order to get an idea of the accuracy of any numerical method.

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Also, we give the concepts of convergence, consistency and stability. It is concerned with the conditions that must be satisfied if the solution of the finite difference equations is to be a reasonably accurate approximation to the solution of the corresponding parabolic or hyperbolic differential equations. It also expresses the descriptive and analytical treatment of convergence. Consistency and stability methods by Von Neumann and matrix form are also discussed.

We describe finally the contribution of different authors who solved the heat equation and hyperbolic wave equation with simple initial and boundary conditions, and then we solved the one-dimensional wave equation with an integral condition and initial and boundary conditions. We develop the numerical methods for obtaining approximate solution of one dimensional wave equation with non-classic boundary conditions. The numerical methods are based on the fully explicit finite difference method, the weighted finite difference method the optimal finite difference method and the implicit finite difference method. These finite difference methods are compared using the largest error terms in the resulting modified equivalent partial differential equation. Stability analysis is discussed in detail by Von Neumann method.

3. Conclusion

In this paper, we give the treatment of non-local boundary conditions and we use the example to support all the finite difference methods which we discussed in chapter 4. We compare the errors of these methods which are written in scientific notation and table shows that optimal finite method is good for the model problem.